1092 Stachys *Canarienſis* fruteſcens, verbaſci folio. *Tourn.*
1093 Tamariſcus latiore folio. *Park.* Germanicus. *Tourn.*
1094 Tamariſcus tenuiore folio. *Park.* Narbonenſis. *Tourn.*
1095 Tacamahac foliis ſerratis. *Plukn. Phyt.* 228. *Fig.* 2.
1096 Tanacetum vulgare luteum. *C. B.*
1097 Tanacetum foliis criſpis. Ibid.
1098 Thlaſpi *Creticum* purpureum. *Park.*
1099 Trachelium umbelliferum *Ponæ*.
1100 Valeriana major hortenſis. *Moriſon.* Phu. *Off.*

XIV. *An Inquiry into the* Meaſure *of the* Force *of Bodies in* Motion: *With a Propoſal of an* Experimentum Crucis, *to decide the Controverſy about it. By* James Jurin, *M. D. Fellow of the* Royal College *of* Phyſicians, London, *and of the* Royal Society.

Read May 30. 1745.

MEchanical Forces may be reduced to two Sorts; one of a Body at Reſt, the other of a Body in Motion.

The Force of a Body at Reſt, is that of a Body lying ſtill upon a Table, or hanging by a Rope, or ſupported upon a Spring, *&c.*

This is called by the Name of Preſſure, Tenſion, Force, or *Vis mortua.*

The *Meaſure* of this Force is the Weight with which the Table is preſſed, or the Rope is ſtretched, or the Spring is bent.

And that *Meaſure* being acknowledged by all Writers, there is no Diſpute about this Sort of Force, notwithſtanding the Diverſity of Appellations by which it is called.

The Force of a Body in Motion is on all hands agreed to be a Power reſiding in that Body, ſo long as it continues its Motion; by means of which it is able to remove Obſtacles lying in its Way; to leſſen, deſtroy, or overcome, the Force of any other moving Body, which meets it in an oppoſite Direction; or to ſurmount any dead Preſſure or Reſiſtance, as Tenſion, Gravity, Friction, *&c.* for ſome time; but which will be leſſen'd or deſtroy'd by ſuch Obſtacles, or by ſuch Reſiſtance, as leſſens or deſtroys the Motion of the Body.

This is called moving Force, *Vis motrix*, and by ſome late Writers, *Vis viva*, to diſtinguiſh it from the *Vis mortua* ſpoken of before: And by theſe Appellations, however different, the ſame Thing is underſtood by all Mathematicians; namely, That Power of diſplacing Obſtacles, withſtanding oppoſite moving Forces, or overcoming any dead Reſiſtance, which reſides in a moving Body, and which, in Whole or in Part, continues to accompany it, ſo long as the Body moves.

But about the *Meaſure* of this Sort of Force, Mathematicians are divided into two Parties: And, in order to ſtate the Caſe fairly between them, it will be neceſſary to ſhew how far the two Parties agree, and in what Point their Diſagreement conſiſts.

Both Sides agree, That the *Meaſure* of this Force depends partly upon the Maſs, or Weight, of the Body, and partly upon the Velocity with which it moves; ſo that, upon any Increaſe either of the Weight, or of the Velocity, the moving Force will become greater.

They alſo agree, That if the Velocity continue the ſame, but the Maſs, or Weight of the Body, be increaſed in any Proportion, the moving Force is increaſed in the ſame Proportion: So that, in this Caſe, the *Meaſure* of the moving Force is the ſame with that of the Weight: Or, when two Bodies move with the ſame Velocity, if the Weight of the ſecond be double, triple, quadruple, of that of the firſt, the moving Force of the ſecond will alſo be double, triple, quadruple, of that of the firſt.

But, when two Bodies are equal, and the Velocities with which they move are different, the two Parties no longer agree about the *Meaſure* of the moving Force.

One Side maintains, That, when the Velocity of the ſecond Body is double, triple, quadruple, of that of the firſt, the *Meaſure* of the moving Force of the ſecond is alſo double, triple, quadruple, of that of the moving Force, being the ſame with that of the Velocity.

The other Side pretend, That, in the ſame Caſe, the moving Force of the ſecond Body is four times, nine times, ſixteen times, as great as that of the firſt; the *Meaſure* of the moving Force being the ſame with that of the Square of the Velocity.

In conſequence of the Agreement in the firſt of theſe two Caſes, and the Diſagreement in the ſecond, the one Side pretends, That the *Meaſure* of the moving Force is, in all Caſes, the Product of the

Weight into the Velocity; and the other, That it is the Product of the Weight into the Square of the Velocity.

This Controverſy was firſt ſtarted by the famous Mr. *Leibnitz*, and has been carried on by him and his Followers for near threeſcore Years; during which Time a great Number of Pieces have been publiſhed on both Sides of the Queſtion, and a great Number of Experiments have been made, or propoſed to be made, in order to decide it. But tho' both Parties agree in the Event of the Experiments, whether actually made, or only propoſed; yet, as the Writers on each Side have found a Way of deducing from thoſe Experiments a Concluſion ſuitable to their own Opinion, the Diſagreement ſtill continues as wide as ever, to the great Scandal of the Learned World.

Now, if we examine carefully into the Reaſon of this, and would ſee by what means it happens, that two oppoſite Concluſions are ſo often drawn from the ſame Experiment, we ſhall find it not ſo much owing to falſe Reaſoning on either Side, (That would be eaſily detected, and ſet right), as to another Cauſe; namely, to their Diſagreement in the Principles upon which the Reaſoning is founded.

For, whereas whatever is laid down on either Side as a Principle, ought to be ſomething all the World agrees in, at leaſt what is admitted by the other Party; without which, all Reaſoning upon it is to no Purpoſe; this Conduct has been ſo little obſerved in the preſent Diſpute, that what has been offered on the one Side as an undoubted Principle or Axiom, has commonly been ſomething that the oppoſite Party does not admit, nay, even abſolutely denies.

Of this it were eaſy to produce a Number of Examples; but I ſhall content myſelf with two only, one taken from each Side.

Thoſe who maintain, That the moving Force is as the Weight into the Velocity, lay down for a Principle, or Axiom, That

When two Bodies meet one another in contrary Directions, if their moving Forces be equal, neither Body will prevail over the other: And if their moving Forces be unequal, the ſtronger will always prevail over the weaker.

This the *Leibnitian* Party deny. They maintain,

That one of theſe Bodies may prevail over the other, though their moving Forces be equal: Nay, that, in many Caſes, the weaker will prevail over the ſtronger.

It is therefore to no Purpoſe to alledge, That the Principle above laid down is founded on common Senſe; or that it was always univerſally received, till this Diſpute began: For, ſince the oppoſite Party now reject it, all Reaſoning upon it can have no Weight with them; you muſt have recourſe to ſomething elſe.

On the other hand, thoſe who adhere to Mr. *Leibnitz*'s Sentiment, lay down for a Principle, That

Equal Effects always ariſe from equal Cauſes; provided the Cauſes be intirely conſumed in producing thoſe Effects.

This their Opponents do not admit, unleſs in the Caſe where thoſe equal Effects are produced in equal Times: And therefore, till both Sides ſhall agree in admitting this Principle, no Argument can be drawn from it by one Party, that will be of any Service to convince the other.

But as this Principle is chiefly made uſe of in reaſoning upon Experiments made with Springs, many of which have been produced by both Parties, in Support of their Opinions, it may be worth while more particularly to conſider, What Right there is on the one Side to impoſe this Principle, and what Reaſons may be given on the other for rejecting it.

When one End of a Spring, wholly unbent, leans againſt an immoveable Support, and the oppoſite End is ſtruck upon by a Body in Motion, which, bending the Spring to ſome certain Degree, does thereby loſe its whole moving Force; the moving Force of the Body may be conſidered as the Cauſe of bending the Spring; and the Bending of the Spring may be looked upon as the Effect of that Cauſe, which is wholly ſpent and conſumed in producing it.

Now if two unequal Bodies, moving with unequal Velocities, ſtrike in this manner upon two equal Springs, and each of them bend the Spring it ſtrikes upon, exactly to the ſame Degree; and by ſo doing, the moving Force of each Body be intirely conſumed; Here, ſay the *Leibnitian* Writers, are two equal Effects produced; for the Springs are equal, and are now equally bent; and the moving Forces, which are the Cauſes of thoſe Effects, are wholly conſumed in producing them; and therefore, by virtue of the Principle above laid down, thoſe Cauſes muſt be equal; that is, the moving Forces of the two Bodies muſt be equal.

But their Antagoniſts will reply, That this Principle is not admitted by them, except the Times of producing thoſe Effects are equal; and that they are not ſo in the Caſe before us: For the greater Body will take up a longer Time in producing its Effect, or in bending its Spring.

If therefore the *Leibnitian* Party pretend, That equal Effects, when produced in unequal Times, do always ariſe from equal Cauſes, they muſt not impoſe this upon their Opponents by way of Principle or Axiom, but muſt demonſtrate it.

Till this be done, there will be Room to doubt, at leaſt, whether the two Bodies have equal moving Forces, though they bend equal Springs to the ſame Degree.

For the larger and ſlower of theſe two Bodies will bend the one Spring more ſlowly; and, conſequently, will be reſiſted for a longer Time, than the ſmaller and ſwifter Body will be reſiſted in bending the other Spring to the ſame Degree.

May not therefore the total Reſiſtance of a Spring be greater, if that Reſiſtance continues for a longer Time?

And, if the total Reſiſtance be greater, muſt not the moving Force, which is deſtroyed and conſumed by that Reſiſtance, be alſo greater?

Is there not Reaſon then to doubt, whether the moving Forces of theſe two Bodies be equal, though they bend equal Springs to the ſame Degree?

In like manner, when a Spring, already bent to ſome certain Degree, does, by unbending, drive before it a Body which gives way to its Preſſure, is there not Room to doubt, whether the Preſſure of the Spring may not produce a greater Effect, when that Preſſure continues for a longer Time?

That Preſſure may be ſaid to produce three Effects, all of which may, if we pleaſe, be conſidered as different from one another.

1. The Preſſure carries the Body thro' a certain Space; by which Space the Length of the bent Spring is increaſed, in returning to its natural Situation.

2. The Preſſure gives to the Body a certain Quantity of Motion.

3. It gives the Body a certain moving Force.

Now, the firſt of theſe Effects is greater, when the Preſſure acts for a longer Time. For, if the Preſſure of a bent Spring, by acting for one Second upon the Body 1, carry that Body 1 thro' the Space 1; the Preſſure of the ſame, or of an equal Spring equally bent, by acting for two Seconds upon the Body 4, will carry that Body 4 thro' the ſame Space 1.

Likewiſe the ſecond Effect is greater, when the Preſſure continues for a longer Time.

For, in the Caſe juſt now mentioned, the Body 4 will have twice the Quantity of Motion that the Body 1 has; though theſe two Quantities of Motion ariſe from the Preſſure of the ſame, or, which is all one, of equal Springs equally bent.

Why therefore are we take it for granted, or to have it impoſed upon us by way of Principle or Axiom, That the third Effect is not greater, when the Time, in which it is produced by the Preſſure of the ſame, or equal Springs, is longer, nay, infinitely longer?

But we are told, that all the Force, which reſided in the Spring, while bent, is now, upon the Unbending of the Spring, communicated to the Body moved. I ask therefore, What was that Force, or what kind of Force was that, which reſided in the Spring, while bent, and without Motion? Was it a bare Preſſure, or a moving Force? A *Vis mortua*, or a *Vis viva?* You muſt acknowledge, it was a *Vis mortua*, a bare Preſſure, and nothing more. But the Force communicated to the Body, and which now reſides in the Body in Motion, is a *Vis viva*, a moving Force. This therefore is not the ſame Force, nor a Force of the ſame kind, as that which reſided in the bent Spring.

It will be ſaid, however, That the Force of the bent Spring is intirely exhauſted in giving the Body its moving Force. I ask therefore again, What is it I am to underſtand by theſe Words, The Force of the Spring is intirely exhauſted? If the Meaning be, that the Spring could not poſſibly give that ſame Body any greater moving Force, than what it has already given, I allow it: But this does not prove, that the ſame Spring, bent afreſh to the ſame Degree, or an equal Spring equally bent, cannot give a greater Force to a greater Body.

But if the Meaning of theſe Words be, That the Spring cannot give a greater moving Force to any Body whatſoever, I muſt anſwer, That this is taking for granted the very Point which is in Diſpute. For the oppoſite Party pretend, That a Body of four Times the Bulk, will receive twice the moving Force in twice the Time, from the Preſſure of the ſame Spring in unbending itſelf, or, if you pleaſe, in exhauſting all its Force.

It is plain, therefore, that the Followers of Mr. *Leibnitz* have no Right to ſay, A Body has ſuch or ſuch a Force, becauſe ſuch or ſuch a Spring has put it in Motion by unbending itſelf, or can be bent by it. This is not a Poſition to be taken for granted, but ſtands in need of a Demonſtration, which nobody has as yet attempted to give, at leaſt from any uncontroverted Principle; and, till this be done, the laying down any ſuch Poſition can have no other Effect, than to perplex the Controverſy more and more, without Hopes of ever coming to an End of it.

For which Reaſon I propoſe to take a quite different Method in what follows, and to lay down nothing, by way of Principle or Axiom, but what is allowed of by all the World, or, at leaſt, has never yet been contradicted *a priori.*

Axiom I.

When a bent Spring does, by unbending itſelf, puſh a Body before it, the greater the Body is, the more ſlowly will the Spring unbend itſelf.

Axiom II.

The more any Spring is bent, the greater is its Preſſure.

Axiom III.

A greater Preſſure produces a greater moving Force, if the Time be given.

Pro-

Propoſition I.

Moving Forces are not proportional to the Maſſes of the Bodies, and the Squares of their Velocities.

Demonſtration.

Let there be two Springs, equal, and equally bent, *A* and *B*, which, by unbending themſelves, puſh before them two unequal Bodies; the Spring *A* puſhing before it the greater Body.

Now, by *Axiom* I. the Spring *A* will unbend more ſlowly than the other: from which it follows, that, at every Inſtant of the Time which the Spring *B* takes up in unbending itſelf, the Spring *A* will have unbent itſelf leſs than *B*, or will be more bent than *B*.

Therefore, by *Axiom* II. the Preſſure of the Spring *A* will, at any Inſtant of that Time, be greater than the Preſſure of the Spring *B* at that ſame Inſtant.

Hence, by *Axiom* III. the naſcent, or infinitely ſmall moving Force, which is produced by the Preſſure of the Spring *A* in every infinitely ſmall Part of that Time, will be greater than that produced by the Preſſure of the Spring *B* in the ſame infinitely ſmall Part of the Time.

Therefore, the Sum of the infinitely ſmall moving Forces; that is to ſay, the whole moving Force, which is produced by the Spring *A*, during that Time, will be greater than the moving Force produced by the Spring *B* in that ſame Time: Or the moving Force of the greater Body will be greater than that of the leſſer, at the Inſtant that the Spring *B*, being now wholly unbent, ceaſes to act any longer upon the Body it has puſhed before it: And as, after that Inſtant, the Spring *A*, not being yet wholly unbent, continues to act upon the greater Body, the moving Force of the greater Body will ſtill continue to increaſe, and conſequently will more and more exceed the moving Force of the ſmaller Body.

But every one knows, that the Products of the Maſſes and Squares of the Velocities are equal in the two Bodies.

Therefore the moving Forces, which we have proved to be unequal, are not proportional to the Products of the Maſſes and Squares of the Velocities. Which was to be demonſtrated.

To conſider this in a particular Example, let us ſuppoſe the Maſſes of the two Bodies expoſed to the Preſſure of the Springs *A* and *B*, to be 4 and 1 reſpectively; and let the Spring *B* unbend itſelf, and thereby give the Body 1 its whole moving Force in one Second of Time. Then, at the End of that Second, the moving Force of the Body 4 will already exceed that of the Body 1, and will ſtill grow greater during another Second of Time. For the Times are as the ſquare Roots of the Maſſes.

Alſo, if the Maſſes be 100 and 1, the moving Force of the Body 100, will, at the End of the firſt Second of Time, be greater than that of the Body 1, and will continue to increaſe during the Space of nine other Seconds.

Corollary. When a bent Spring does, by unbending itſelf, drive a Body before it, the larger that Body is, the greater will be the moving Force which it receives from the Spring.

Having now clearly proved, that the moving Forces are not proportional to the Squares of the Velocities, I proceed next to demonſtrate, that they are proportional to the Velocities themſelves: And, in order thereto, I ſhall, as I have hitherto done, make uſe of no other Principles or Axioms, than ſuch as are admitted on both Sides, or, at leaſt, have never yet been controverted *a priori* by either Party.

Axiom IV.

Springs of unequal Lengths, when bent alike, have equal Preſſures.

We ſpeak here of Springs equal in all Reſpects, except the Length only; and, by being bent alike, we mean, that they are ſo compreſſed, as that the Lengths they are now reduced to, are exactly proportional to their natural Lengths, or to the Lengths they are of when no way compreſſed.

In this Condition, if one be directly oppoſed to the other, they will mutually ſuſtain each other's Preſſure, ſo as to maintein a perfect *Æquilibrium*: Or, if each be placed ſeparately in a vertical Situation, they will ſuſtain equal Weights. And in one or the other of theſe Caſes, it is evident, that they muſt exerciſe equal Preſſures.

Axiom V.

Equal Preſſures in equal Times produce equal moving Forces.

Propoſition II.

Moving Forces are proportional to the Maſſes and Velocities jointly.

Demonſtration.

Let there be two Springs, of the Lengths 1 and 2, but equal in all other Reſpects, and bent alike: And, in unbending themſelves, let the Spring 1 drive before it a Body whoſe Maſs is 2; and the Spring 2 another Body of the Maſs 1.

Now, by *Coroll.* 11. of my general Theorem concerning the Action of Springs, theſe two Springs will unbend themſelves exactly in the ſame Time; and, conſequently, the Spring 2 will unbend itſelf with a Velocity double of that of the Spring 1: And. by *Coroll.* 12. of the ſame Theorem, it will give to the Body 1 a Velocity double of that, which the Body 2 will receive from the Spring 1.

Alſo, as the two Springs were ſuppoſed to be bent alike at firſt, and the Spring 2 unbends itſelf with a Velocity double to that of the Spring 1, it is manifeſt, that, during the whole Time of their Expanſion, they will be always bent alike, one to the other.

Therefore, by *Axiom* IV. their Preſſures will be conſtantly equal one to the other: And hence, by *Axiom* V. the infinitely ſmall moving Forces produced by each of theſe Springs, in every infinitely ſmall Part of Time, will be equal one to the other. Conſequently, the Sums of thoſe infinitely ſmall moving Forces, that is, the whole moving Forces, produced by the two Springs, will be equal one to the other. And the Maſſes of the two Bodies being 2 and 1, and their Velocities being 1 and 2 reſpectively, it is plain, that the moving Forces are proportional to the Maſſes and Velocities jointly. Which was to be demonſtrated.

For the greater Facility of examining this Demonſtration, we have ſuited it to a ſingle Caſe only, and that the moſt ſimple that can be ſuppoſed: But every body will ſee, how eaſy it is to form a general one upon the ſame Principles.

As we do not think, that any Flaw can be found in either of the Demonſtrations above laid down; and the Axioms, upon which they are founded, have never yet been diſputed, as far as we know; we preſume, that the *Leibnitian* Opinion about the Meaſure of moving Forces, is inconteſtably overthrown by the firſt Propoſition, and the oppoſite Sentiment is as evidently eſtabliſhed by the ſecond.

But, if any Reader ſhall be of a different Opinion, we muſt beg Leave to propoſe to his Conſideration the following Experiment, which we hope may juſtly deſerve the Name of an *Experimentum Crucis*; and, as ſuch, may put a final Period to this Controverſy.

It is not new indeed, having been propoſed before by myſelf and others; but, as the Manner, in which it was formerly offered, has given Occaſion to ſome Objections, which, tho' not affecting the Subſtance of the Argument drawn from it, may yet have amuſed and embaraſſed the leſs attentive Readers, I ſhall now propoſe it in ſuch a Manner, as may obviate all thoſe Difficulties, and, I think, will render it abſolutely deciſive. To me, I am ſure, it will be ſo, ſince I ſhall immediately embrace the

Leibnitian Doctrine, if my Argument drawn from it can receive a clear and ſatisfactory Anſwer.

Experiment.

Upon an horizontal Plane, at Reſt, but moveable with the leaſt Force, ſuppoſe upon a Boat in a ſtagnant Water, let there be placed, between two equal Bodies, a bent Spring, by the unbending of which the two Bodies may be puſhed contrary Ways.

In this Caſe it is evident, that the Velocities, which the two Bodies receive from the Spring, will be exactly equal, and their moving Forces will alſo be exactly equal; and that the Plane they move upon, and alſo the Boat upon which it lies, will have no Motion given them either Way.. Let us call the Velocity of each Body 1, and the moving Force alſo 1.

Now, let us ſuppoſe the Spring to be bent afreſh to the ſame Degree as before, and to be again placed between the two Bodies lying at Reſt; then let the Plane, upon which the Spring and the Bodies lie, be carried uniformly forwards, in the Direction of the Length of the Spring, with this ſame Velocity 1. In this Caſe it is manifeſt, that each of the Bodies will have the Velocity 1, and the moving Force 1, both in the Direction of the *Axis* of the Spring.

During this Motion, let the Spring again unbend, and puſh the two Bodies contrary Ways, as before, the one forwards, the other backwards: Then the Spring will give to each of theſe Bodies the Velocity 1, as before, when the Plane was at Reſt.

By this means the hindmoſt Body, or that which is puſhed backwards, will have its Velocity 1, which it had before by the Motion of the Plane, now intirely deſtroy'd, and will be abſolutely at Reſt.

But the Body, which is puſhed forwards, will now have the Velocity 2, namely 1 from the Motion of the Plane, and 1 from the Action of the Spring.

Thus far every body agrees in what will be the Event of this Experiment.

But the Queſtion is, What will be the moving Force of the foremoſt Body, or of that which is puſhed forwards, and which has the Velocity 2; *viz.* 1 from the Motion of the Plane, and 1 from the Action of the Spring.

By the *Leibnitian* Doctrine, its moving Force muſt be 4: And, if ſo, it muſt have received the moving Force 3 from the Action of the Spring; for it had only the moving Force 1 from the Motion of the Plane.

Let us examine, whether this be poſſible, or reconcileable to their own Doctrine.

Their Doctrine is, That equal Springs, equally bent, will, by unbending themſelves, give equal moving Forces to the Bodies they act upon, whatever thoſe Bodies are.

We agree to this, not generally indeed; but in the Caſe before us, where the Bodies are of equal Maſſes or Weights, we agree to it.

Let us therefore imagine the bent Spring, which is placed between the two Bodies, to be divided tranſverſly into two equal Parts. In this Caſe it is plain, that the two Halves of the Spring, may be conſidered as two intire Springs, equal, and equally bent, each of which reſts at one End in *Æquilibrio* againſt the other Spring, and at the oppoſite End, preſſes againſt the Body it is to move.

Con-

Conſequently, by the *Leibnitian* Doctrine, to which, in this particular Caſe, where the Bodies are equal, we alſo agree, the two Springs will give equal moving Forces to the two Bodies.

But the moving Force received by the hindmoſt Body from the hinder Spring, was undoubtedly the moving Force 1: For by that Force given it in the Direction backwards, the moving Force 1, which it had before from the Motion of the Plane in the Direction forwards, is exactly balanced and deſtroyed, the Body remaining, as was obſerved before, in abſolute Reſt.

Therefore the moving Force received by the foremoſt Body from the foremoſt Spring, was alſo the moving Force 1. And this, added to the other moving Force 1, which it had before from the Motion of the Plane, makes the moving Force 2, and not the moving Force 4, as the *Leibnitian* Philoſophers pretend.

Conſequently, that Body, which had before the Velocity 1, and the moving Force 1, and now has the Velocity 2, has alſo the moving Force 2: That is, the moving Forces are proportional to the Velocities.